Hypercomplex Seminar 2021

November 11–14

Program and abstracts

(2021-11-04)

November 11, 2021

Opening ceremony (9:00-9:50)

Session A1 (A tribute to the works of Julian Ławrynowicz),
Chairmen: Dariusz Partyka and Mariusz Zubert

10:00-10:50 *Osamu Suzuki*, (→)
Non-commutative Galois theory and its application to general evolution

11:00-11:50 *Massimo Vaccaro*, (→)
$Sp(n)$-orbits of isoclinic subspaces in the real Grassmannians.

Session A2 (Monogenic functions),
Chairman: Massimo Vaccaro

12:30-13:20 *Sergiy Plaksa*, (→)
Monogenic functions in a harmonic algebra

13:30-13:55 *Serhii Gryshchuk*, (→)
Monogenic functions with values in commutative complex algebras of the second rank with unity and generalized biharmonic equation with simple characteristics

14:00-14:25 *Vitalii Shpakivskyi*, (→)
Hypercomplex method for solving linear PDEs

Session A3 (Hypercomplex structures),
Chairman: Sergiy Plaksa

15:00-15:50 *Lino F. Reséndis Ocampo*, (→)
Bicomplex Bergman Projection

16:00-16:25 *M. Elena Luna-Elizarrarás*, (→)
Hyperbolic curves and integration of bicomplex functions

16:30-17:00 *Valery Volchkov*, *Vitaly Volchkov*, (→)
Mean periodicity on the Cayley projective plane

Session A4 (Complex analysis of several variables),
Chairman: Lino F. Reséndis Ocampo

17:30-17:55 *Piotr Liczberski*, (→)
Past and present extremal problems for series of holomorphic functions of several variables

18:00-18:25 *Renata Długosz*, *Piotr Liczberski*, *Edyta Trybucka*, (→)
The Fekete-Szegö estimates of homogeneous polynomials for functions from some Bavrin's families

18:30-18:55 *Anna Kimaczyńska*, (→)
Morse Lemma

19:00-19:25 *Tetiana Osipchuk*, (→)
On lineal convexity generalized to commutative and non-commutative algebras

November 12, 2021

Session B1 (Condensed matter physics),
Chairman: Łukasz T. Stępień

9:00-9:50 *Zhidong Zhang*, (→)
Exact solution of ferromagnetic 3D Ising model and computational complexity of spin-glass 3D Ising system

10:00-10:25 *Dariusz Sztenkiel*, *K. Gas*, *M. Foltyn*, *N. Gonzalez Szwacki*, *J. Domagała*, *D. Hommel*, *T. Dietl*, *M. Sawicki*, (→)
Atomistic spin model simulations of magnetic and magneto-electric properties of ferromagnetic (Ga,Mn)N

10:30-10:55 *Yadhu Krishnan Edathumkandy*, *Maciej Sawicki*, *Dariusz Sztenkiel*, (→)
Comparative study of magnetic properties of Mn^{3+} magnetic clusters in GaN using classical and quantum mechanical approach

Session B2 (Ontology of quantum mechanics),
Chairman: Adam Paszkiewicz

11:30-12:20 *Chantal Roth*, *Marek Danielewski*, (→)
Quaternion Quantum Mechanics: from Hamilton spacetime to the displacement four-potential in the Planck-Kleinert crystal

12:30-13:20 *Manfried Faber*, (→)
A geometric model in 4D space-time for electrodynamic phenomena

13:30-14:20 *Jarosław Duda*, (→)
Exploring resemblance between liquid crystal topological defects and particle physics

14:30-14:55 *Adam Paszkiewicz*, *Stanisław Goldstein*, [*Chairman*: Mariusz Zubert] (→)
Linear combinations of projections in von Neumann factors

15:00-15:50 *Jacek Leśkow*, [*Chairman*: Mariusz Zubert] (→)
Cyclostationary Time Series. Functional Data Analysis and Time Series.

Session B3 (Theory of defects in topology of order parameter),
Chairman: Marek Danielewski

16:30-17:20 *Ilan Roth*, (→)
Topological Evolution of Magnetically Knotted-Linked Physical Systems

17:30-17:55 *Oleh Yermakov*, (→)
Mathematical approaches and structures in photonic systems

18:00-18:25 *Sabri Koraltan*, *Michael Heigl*, *Claas Abert*, *Manfred Albrecht*, *Dieter Suess*, (→)
Skyrmions and antiskyrmions in ferrimagnetic multilayers

18:30-18:55 *Łukasz T. Stępień*, (→)
On some exact solutions of certain nonlinear PDE's in physics

19:00-19:25 *Vasileios Vachtsevanos*, (→)
Statistics of topological defects in nanostructures based on the Kibble-Zurek Mechanism

Round Table Discussion I (20:00-21:00)
Chairman: Ilan Roth

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Session C1 (Embodied AI and evolutionary systems),
Chairman: Mariusz Zubert

9:00-9:50 *Paulius Miškinis*, (→)
About the class of completely integrable evolutionary models

10:00-10:25 *Fabio Bonsignorio*, (→)
Towards a New Paradigm Embodied AI unifying Robust and Adaptive Soft Robotics and AI

10:30-10:55 *Tomasz Talarczyk*, *Marcin Morawski*, *Marcin Malec*, (→)
Depth control for biomimetic and hybrid unmanned underwater vehicles

Session C2 (Mathematical modeling in Physics and Computer Science),
Chairman: Fabio Bonsignorio

11:30-12:20 *Maciej Jaworski*, (→)
Data stream mining algorithms – mathematical perspective

12:30-12:55 *Alexander Chichurin*, *Galina Filipuk*, (→)
An algorithm for finding solutions of a second-order nonlinear differential equation expressed in terms of the Mathieu functions

13:00-13:50 *Marta Dudek*, *Janusz Garecki*, (→)
Riemannian structure imposed on Friedmann and more general spacetimes

Session C3 (Data analysis and machine learning),
Chairman: Maciej Jaworski

14:30-15:20 *Radosław A. Kycia*, (→)
Clusterization in action

15:30-15:55 *Mateusz Chmurski*, *Mariusz Zubert*, *Avik Santra*, (→)
Study of Edge-Optimized Deep Learning Classifiers for Radar-Based Gesture Recognition

16:00-16:25 *Dana Simian*, (→)
Methods for Parameters Optimization in SVMs and SVRs

Session A6 (The inner radii problems),
Chairman: Serhii Gryshchuk

17:00-17:25 *Liudmyla Vyhivska*, (→)
Inequalities for the inner radii of non-overlapping symmetric domains

17:30-17:55 *Iryna Denega*, (→)
Estimates of the products of inner radii of mutually non-overlapping domains

18:00-18:25 *Yaroslav Zabolotnyi*, (→)
Estimates of products of some powers of inner radii for multiconnected domains

Session A7 (Poster session),
Chairman: Armen Grigoryan

19:00-19:50 *Ramojus Balevičius*, *Paulius Miškinis*, (→)
Experimental evaluation and theoretical modeling of air viscosity related parameters

19:00-19:50 *Jacek Dziok*, (→)
Classes of meromorphic harmonic functions defined by Sălăgean operator

19:00-19:50 *Zoya Eremenko*, (→)
Resonant frequencies in microwave spheroid cavity resonator with small eccentricity using the local point method

19:00-19:50 *Olena Karupu*, (→)
On some finite difference properties of analytic functions

19:00-19:50 *Bogdan Klishchuk*, *Ruslan Salimov*, (→)
On the order of growth of one class of homeomorphisms

19:00-19:50 *Viktor Savchuk*, *Olga Rovenska*, (→)
Approximation of some class of holomorphic functions by Cesàro means

19:00-19:50 *Viktor Savchuk*, *Olga Rovenska*, (→)
Approximation of classes of harmonic bounded functions by Fejer means

19:00-19:50 *Natalia Zorii*, (→)
Harmonic measure for the fractional Laplacian and its applications

Round Table Discussion II (20:30-21:00)
Chairman: Sergiy Plaksa

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Session D1 (Superconducting and semiconductor electronics),
Chairman: Khrystyna Gnatenko

9:00-9:50 *Mikhail Belogolovskii*, (→)
Multilayered Josephson junctions for quantum sensing

10:00-10:25 *B. Stojewski*, *Krzysztof Pomorski*, (→)
Fundamental description of Wannier qubits of any topology in semiconductor by analytical and numerical computations

10:30-10:55 *Łukasz Pluszyński*, *Krzysztof Pomorski*, (→)
Towards analog electronic simulation of electrostatically interacting quantum particles

Session D2 (Physics of superconducting materials),
Chairman: Mikhail Belogolovskii

11:30-12:20 *Catherine Pépin*, (→)
Charge order and Strange metals in cuprate superconductors

12:30-12:55 *Marcin Kowalik*, *R. Zalecki*, *M. Giebułtowski*, *J. Niewolski*, *W. Tokarz*, *S. Wolski*, (→)
The application of unsupervised learning to the AC susceptibility measurements of High-Temperature Superconductors

13:00-13:25 *Marek Giebułtowski*, (→)
Experimental high-temperature superconductors study in direction of application in information processing technologies

Session D3 (Experiments with superconducting and seminconductor electronics),
Chairman: Catherine Pépin

14:00-14:50 *Khrystyna Gnatenko*, (→)
Studies of properties of spin systems on a quantum computer

15:00-15:25 *Krzysztof Pomorski*, (→)
Fundamental description of Wannier qubits of any topology in semiconductor by analytical and numerical computations

15:30-15:55 *Krzysztof Pomorski*, *Maksymilian Siembab*, (→)
Modeling of electrostatic programmable quantum gates of different topologies

16:00-16:25 *Jakub Mojsiejuk*, (→)
CMTJ – Simulation tool for multilayer spintronics devices

Session A8 (Complex and real analysis of one variable),
Chairman: Radosław Kycia

17:00-17:25 *Anna Futa*, *Dariusz Partyka*, (→)
The Schwarz type inequalities for harmonic functions of the unit disk into itself normalized on the boundary by a finite collection of arcs

17:30-17:55 *Armen Grigoryan*, (→)
Bounded biharmonic mappings

18:00-18:25 *Maciej Parol*, (→)
On the Koebe Quarter Theorem for certain polynomials

18:30-18:55 *Ruslan Salimov*, *Mariia Stefanchuk*, (→)
Local Properties of Solutions of Nonlinear Beltrami Equation

19:00-19:25 *Anatoly Serdyuk*, *Tetiana Stepanyuk*, (→)
Asymptotically best possible Lebesgue-type inequalities for the Fourier sums on the sets $C_\beta^{\alpha,r}L_p$

19:30-19:55 *Jacek Marchwicki*, (→)
Guthrie and Nymann Cantorval

Closing ceremony (20:00-20:15)

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Experimental evaluation and theoretical modeling of air viscosity related parameters

Abstract. Experimental measurements of the constant speed of falling bodies of small mass near the surface of the earth made it possible to estimate the value of the dynamic viscosity of air. More accurate measurements were carried out on the bench, measuring the rate of water flowing out of the vessel. Based on these measurements, the kinematic coefficient of air viscosity, the mean free path and the mean square thermal velocity of air molecules were determined. Measurements of the temperature dependence of the coefficient of dynamic viscosity of air made it possible to determine the Sutherland corrections for the size of air molecules and other related quantities. The obtained experimental data are in good agreement with the known theoretical values given in the literature.

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Multilayered Josephson junctions for quantum sensing

Abstract. Developing a nanoscale magnetic sensing technique applicable to individual molecules or magnetic nanoparticles could enable revolutionary advances in all spheres of life. Unfortunately, in the case of nanoSQUIDs, the dc SQUID loop may not satisfy the assumption to be thicker than the magnetic penetration depth and the critical current of the loop will approach that of the junctions, implying no distinction between a single Josephson device and a dc SQUID. If so, it is quite possible to use a single junction as a nanometer-scale ultra-sensitive and ultra-efficient magnetic field detector.

In the contribution, I present studies of quantum heterostructures based on a ferromagnetic (F)/normal (N) metal multilayer proximitized by superconducting electrodes to form a novel Josephson weak-link device highly sensitive to magnetic fields. The device is a single Josephson junction, which manifests quasi-sinusoidal critical current oscillations resembling a SQUID response. Another striking feature of our devices involving F/N multilayers is the background Josephson current on which the oscillations are superposed.

I argue that the SQUID-like oscillations in the critical current vs magnetic field dependences arise due to the dephasing accumulated by an electron (or a hole) during its chaotic motion across the weak link. The peculiarity of the FN multilayers is that the diffusion coefficient for the charge transport perpendicular to the metallic layers is much smaller than being parallel to them. As a result, for a relatively large weak link, the current in its central part is strongly suppressed relative to the edges. It leads to the localization of the supercurrent to the edges. The background current may appear due to the difference in transmission eigenvalues for two edges of the weak link, leading to the asymmetric Josephson effect.

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Joint work with: I.P. Nevirkovets and J.B. Ketterson, Northwestern University, Evanston, USA

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Towards a New Paradigm Embodied AI unifying Robust and Adaptive Soft Robotics and AI

Abstract. In the coming decades we will need to dramatically increase work productivity not only to cope with a shrinking work-force and growing number of people in old and very old age in developed countries, but also to mobilize resources to help the ecologically sustainable development of the global economy and provide food and infrastructure to billions of more people. A steep progress in Robotics and AI seems a dramatic necessity in this context. Luckily, the current wave of innovation in Robotics, integrating Machine Learning, Probabilistic Robotics, and some AI is already having significant impact on our economy and our society.

However, it is a widespread opinion that Robotics still need much more robustness, safety, lower manufacturing costs, and reduced control complexity and effort; while it aims to more and more complex and adaptive behaviors in open ended environments.

A foundational approach to Embodied AI, disrupting both Robotics and AI, is often referred to as 'Morphological Computation', i.e., the outsourcing of computation from the controller to body-environment interactions of the system. The concept is often used to describe rather different phenomena in the literature. While there seems to be a consensus about the importance of embodiment, there is still no clear definition of how the embodiment of an agent – and typically of an intelligent robot – should actually be defined. Many of the conceptual definitions of embodiment that have been proposed so far do not provide much more than the common sense understanding of embodiment, which is why some researchers believe that an operational and quantitative approach – i.e. 'Morphological Computation' is important.

There are already different approaches to quantify embodiment. Examples from the field of information theory have been developed by the author and by a handful of other researchers.

The talk will review the existing different concepts and quantifications of embodiment and will show how they overlap and differ, thereby leading to a better understanding and a clearer picture of what actually is meant as 'embodiment' and 'Morphological Computation'.

It is thought that a clearer understanding of the basic quantitative physical aspects of embodiment may pave the way to radically new and significantly more effective approaches to the modeling and control of intelligent robots (with rigid and soft body parts) perceiving and acting stochastically in unstructured and partially known open ended environments.

Moreover, the research methodology requires to be improved if we want to ground paradigm and modeling choices on experimental evidence.

To what extent, under which conditions — and critically in which timeframe — will it be possible to achieve the necessary advancements to be able to exploit robots to dramatically increase productivity and for elder care? Which mathematical challenges will arise? Are those bottlenecks to AI and robotics development?

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An algorithm for finding solutions of a second-order nonlinear differential equation expressed in terms of the Mathieu functions

Abstract.

We present a method to solve nonlinear differential equations via the Schwarzian derivative reduction to linear differential equations and via one more reduction to the Ermakov-Painleve XXV equation. We show that the general solution of second order nonlinear differential equations is expressed in terms of Mathieu functions.

The algorithm of finding classical solutions for nonlinear second-order differential equations allowing linearization by using the Schwarzian derivative is presented. Implementation of this algorithm is shown in examples. It is also shown how to integrate such equations when the solutions of the corresponding linear differential equations of the third order in special functions (Mathieu functions) are known. The linear differential equation

$$y''' + (a\cos(2x) + b)y' - a\sin(2x)y = 0 \ (a, b \text{ are constants}),$$

has the general solution of the form

$$y = c_1 \text{MathieuC}^2\left[\frac{b}{4}, -\frac{a}{8}, x\right] + c_2 \text{MathieuC}\left[\frac{b}{4}, -\frac{a}{8}, x\right] \text{MathieuS}\left[\frac{b}{4}, -\frac{a}{8}, x\right] + c_3 \text{MathieuS}^2\left[\frac{b}{4}, -\frac{a}{8}, x\right],$$

where c_1, c_2, c_3 are arbitrary constants. The corresponding nonlinear differential equation

$$(12w - 6(a\cos(2x) + b))\, w'' - 15w'^2 - 30a\sin(2x)w' + 8w^3 - 24(a\cos(2x) + b)w^2 +$$
$$\left(18(a\cos(2x) + b)^2 + 24a\cos(2x)\right) w - 4(a\cos(2x) + b)^3 - 12a\cos(2x)(a\cos(2x) + b) - 15a^2\sin^2(2x) = 0$$

has the general solution of the form

$$w = \frac{1}{2}\Bigg(\frac{864C_1^2}{\text{MathieuC}\left[\frac{b}{4}, -\frac{a}{8}, x\right]^4 \left(72C_1^2\left(\int \frac{1}{\text{MathieuC}\left[\frac{b}{4}, -\frac{a}{8}, x\right]^2} dx + C_2\right)^2 + 1\right)^2} + a\cos(2x) + b\Bigg),$$

where C_1, C_2 are arbitrary constants. The theorem about the structure of the general solution of the second-order nonlinear differential equation is proved.

Keywords and phrases: Computer algorithm, Schwarzian derivative, Mathieu functions, second order nonlinear differential equation, generalized Riccati equation

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Study of Edge-Optimized Deep Learning Classifiers for Radar-Based Gesture Recognition

Abstract. The increasing importance of technology in our daily lives leads to the need for the development of more convenient methods of human-computer interaction (HCI). Given the limitations of existing HCI methods, hand gesture recognition-based HCI may serve as a more intuitive mode of human-machine interaction. In addition to that, the system should be able to operate in various environments, independently from lighting conditions, various backgrounds and surrounding noise factors. Moreover, the system has to be deployable on low-power devices in order to be applicable in broadly defined Internet of Things (IoT) and smart home devices. Recent advances of deep learning exhibit potential in gesture recognition [1]. However, the complexity of the recent deep learning models forces the usage of high-performance hardware. The embedded platforms are constrained in terms of available resources. This work addresses the aforementioned problems. First of all, we propose an alternative radar signal processing pipeline, allowing for avoidance of recurrent structures and inefficient topological design. Secondly, we propose the dedicated neural network classifier and compare it with the state-of-the-art classifiers. The proposed classifier has been optimized for the Intel Neural Compute Stick 2 (Intel NCS2) and achieves promising classification accuracy of 97.57%.

Keywords and phrases: Accelerator,data augmentation,edge computing,FMCW,HCI,gesture recognition,neural networks,DNNs,optimization,radar,intel NCS2

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Estimates of the products of inner radii of mutually non-overlapping domains

Abstract. Let $\mathbb{N}$ be the set of natural numbers, let $\mathbb{C}$ be the complex plane, and let $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be its one-point compactification. A function $g_B(z,a)$ which is continuous in $\overline{\mathbb{C}}$, harmonic in $B\backslash a$ apart from z, vanishes outside B, and in the neighborhood of a has the following asymptotic expansion

$$g_B(z,a) = -\ln|z-a| + \delta + o(1), \quad z \to a,$$

is called the (classical) Green function of the domain B with pole at $a \in B$. For $a = \infty$ we define the function $g_B(z,\infty)$ in a similar way $g_B(z,\infty) = \ln|z| + \delta + o(1)$, $z \to \infty$. The inner radius $r(B,a)$ of the domain B with respect to a point a is the quantity e^{δ} (see, for example, [1,2]).

The goal of the present work is to get the upper bounds for functionals of the following form for all values of the parameter $\gamma \in (0,n]$

$$I_n(\gamma) = r^{\gamma}\left(B_0, 0\right) \prod_{k=1}^{n} r\left(B_k, a_k\right),$$

$$Y_n(\gamma) = r^{\gamma}\left(B_\infty, \infty\right) \prod_{k=1}^{n} r\left(B_k, a_k\right),$$

where $n \in \mathbb{N}$, $n \geqslant 2$, $\{a_k\}_{k=1}^{n}$ is an arbitrary fixed system of points, B_0, B_∞, B_k is any system of mutually non-overlapping domains such that $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, $\infty \in B_\infty \subset \overline{\mathbb{C}}$, $a_k \in B_k \subset \overline{\mathbb{C}}$ for $k = \overline{1,n}$. The following propositions hold.

Theorem 1. Let $n \in \mathbb{N}$, $n \geqslant 2$, $\gamma \in (0,\, n]$. Then, for any fixed system of points $\{a_k\}_{k=1}^{n} \subset \mathbb{C}\backslash\{0\}$ and for any collection of mutually non-overlapping domains B_0, B_k, $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1,n}$, the inequality

$$I_n(\gamma) \leqslant n^{-\frac{\gamma}{2}} \left(I_n(0)\right)^{1-\frac{\gamma}{n}} \left(\prod_{k=1}^{n} |a_k|\right)^{\frac{2\gamma}{n}}$$

is satisfied.

Theorem 2. Let $n \in \mathbb{N}$, $n \geqslant 2$, $\gamma \in (0,\, n]$. Then, for any fixed system of points $\{a_k\}_{k=1}^{n} \subset \mathbb{C}$ and for any collection of mutually non-overlapping domains B_∞, B_k, $\infty \in B_\infty \subset \overline{\mathbb{C}}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1,n}$, the inequality

$$Y_n(\gamma) \leqslant n^{-\frac{\gamma}{2}} \left(Y_n(0)\right)^{1-\frac{\gamma}{n}}$$

is valid.

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Keywords and phrases: the Green function, inner radius of the domain, mutually non-overlapping domains

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The Fekete-Szegö estimates of homogeneous polynomials for functions from some Bavrin's families

Abstract. In the lecture will be considered a generalization of the well-known Fekete-Szegö [FS] type problem onto some Bavrin's families $\mathcal{M}_{\mathcal{G}}$, $\mathcal{N}_{\mathcal{G}}$ [Bav] and $\mathcal{K}_{\mathcal{G}}^{-}$ [LL] of complex value holomorphic functions of several variables. The definitions of Bavrin's families $\mathcal{M}_{\mathcal{G}}$, $\mathcal{N}_{\mathcal{G}}$ correspond to geometric properties of univalent functions of a complex variable, like as starlikeness and convexity. Similarly the family $\mathcal{K}_{\mathcal{G}}^{-}$ corresponds to the class $\mathcal{K}^{s}$ of functions of one complex variable introduced by Gao and Zhou [GZ]. Most of the results presented are from the paper [DL1], [DL2].

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Exploring resemblance between liquid crystal topological defects and particle physics

Abstract. There are experimentally observed long-range e.g. Coulomb-like interactions for topological defects in liquid crystals, suggesting investigation how far can we take this resemblance with particle physics. I will discuss postulating skyrmion-like Lagrangian to get electromagnetism for their effective dynamics, interpreting filed curvature as electric field - making Gauss law count (quantized) topological charge. For biaxial nematic - with 3 distinguished axes, hedgehogs of one of 3 axes are different mass realizations of the same topological charge - resembling 3 leptons. Further baryon-like topological structures require charge, which has to be compensated for neutron - suggesting why it is heavier than proton. For analog of quantum phase there is derived Klein-Gordon-like equation

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Riemannian structure imposed on Friedmann and more general spacetimes

Abstract. In the paper we consider two Finsler-like Riemannian metrics, which can be in a natural way introduced into general relativity. One of those metrics γ_{ab} is degenerate and the second h_{ab} is nondegenerate. We are mainly interested with the metric h_{ab} and comparing the geometric structure determined by this metric h_{ab} with the geometric structure determined by the Lorentzian metric g_{ab} of the underlying spacetime. Full comparison we have given for Friedmann Universis.

The preliminary version of the paper was presented by one of us (J.G.) on conference POTOR 6 in Szczecin 2019. We think that the our introduction of the Riemannian metric h_{ab} into spacetime is simpler and more general than to so-called Wick rotation.

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Classes of meromorphic harmonic functions defined by Sălăgean operator

Abstract. A continuous function $f = u + iv$ is a complex valued harmonic function in a domain D if both u and v are real harmonic in D. If D is the exterior of the unit disc i.e. $\mathbb{D} := \{z \in \mathbb{C} : |z| > 1\}$, then we say that f is meromorphic harmonic function. Hengartner and Schober [1] showed that meromorphic harmonic, orientation preserving, univalent mapping f, satisfying $f(\infty) = \infty$, must admit the representation $f(z) = h(z) + \overline{g(z)} + A\log|z|$, where $A \in \mathbb{C}$,

$$h(z) = az + \sum_{n=1}^{\infty} a_n z^{-n}, \quad g(z) = bz + \sum_{n=1}^{\infty} b_n z^{-n} \quad (z \in \mathbb{D},\ 0 \leq |a_0| < |b_0|),$$

and $\overline{f_{\overline{z}}}/f_z$ is analytic and bounded by 1 in $\mathbb{D}$. By using the Sălăgean operator [2] we define and study some classes of meromorphic harmonic functions. We give necessary and sufficient conditions, some topological properties and extreme points for these classes of functions. Moreover, by using extreme points theory we also obtain coefficients estimates, distortion theorems and integral mean inequalities.

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Resonant frequencies in microwave spheroid cavity resonator with small eccentricity using the local point method

Abstract. We have developed a new method for solving electrodynamic tasks such as eigenfrequencies in electromagnetic cavities for non-integrated systems such as a spheroidal resonator in spherical coordinate. The main feature of our method is the ability to satisfy the boundary conditions at the interfaces between the media at individual points located on these boundaries. At first we carried out the numerical study to find resonant frequencies for small eccentricity for the spheroidal resonator, but our method can be used for any values of the eccentricity. We have compared our results with other similar methods and obtained excellent agreement. These methods were the perturbation method and the spheroidal eigenfunction method.

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A geometric model in 4D space-time for electrodynamic phenomena

Abstract. With three rotational degrees of freedom of spatial Dreibeins and an appropriate Lagrangian [1,4] we describe electromagnetic phenomena [2]. Stable solitonic excitations we compare with the lightest fundamental electric charges, electrons and positrons. Two Goldstone bosons we relate to the properties of photons [3]. These excitations are characterised by three topological quantum numbers, corresponding to charge, spin and photon number.

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The Schwarz type inequalities for harmonic functions of the unit disk into itself normalized on the boundary by a finite collection of arcs

Abstract. Given $n \in \mathbb{N}$ let $T_1, T_2, \ldots, T_n$ be closed arcs contained in the unit circle $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$, with positive length, total length 2π and covering $\mathbb{T}$. Write $\mathcal{F}$ for the class of all complex-valued harmonic functions of the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ into itself satisfying the following condition: For each $k \in \{1, 2, \ldots, n\}$ and for almost every $z \in T_k$ the radial limit of the function F at the point z belongs to the convex hull spanned by the origin and arc T_k. The leading problem is to estimate $|F(z)|$ for $F \in \mathcal{F}$ and $z \in \mathbb{D}$. The symmetric case of three arcs T_1, T_2, T_3 of the same length was studied in [2]. In [1] a more generalized case were considered where $n \geq 3$ and each arc T_k is of the length less than π. This presentation is a short summary of recent results in this direction, obtained without limiting the length of the arcs.

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Experimental high-temperature superconductors study in direction of application in information processing technologies

Abstract. Superconductors can be used in implementation of Josephson junction based classical and quantum electronics. Properties of Josephson junction are encoded in non-linear DC and AC current-voltage (I-V) characteristics. The first step towards obtaining sophisticated circuits is relying on the syntheses and characterisation of bulk superconducting material. Various experimental technique study were conducted by measurements of resistance, AC susceptibility, XRD pattern, critical current characterization and finally by scanning electron microscope sample characterization. The studied material was $Tl_2Ba_2Ca_2Cu_3O_x$ (Tl-2223) high temperature bulk superconductor. Resistance and AC susceptibility are predetermining critical exponents which are depending on the temperature. Critical currents and temperatures are presented in function of sample oxygenation temperature.

Keywords and phrases: thallium cuprate

Joint work with: Marcin Kowalik, Maria Rajska, Waldemar Tokarz, Ryszard Zalecki, Łukasz Gondek

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Studies of properties of spin systems on a quantum computer

Abstract. Methods for detecting energy levels of spin systems on a quantum computer based on the studies of evolution of a physical quantities are discussed [1, 2]. It is shown that studies of evolution of mean value of an operator anticommuting with a Hamiltonian of a system allow quantifying its spectrum. Namely the Fourier transformation of the time dependence has delta-picks corresponding to the energy levels [1]. It is worth noting that in the case of existence of operator anticommuting with a Hamiltonian the energy spectrum of a system is symmetric with respect to transformation $E \to -E$. Energy spectra that do not posses such symmetry can be detected studying the evolution of a probe spin [2]. The results of quantifying of the energy levels of spin systems (spin in magnetic field, spin chain in magnetic field, Ising model on squared lattice) with the proposed methods on IBM's quantum computers (ibmq-manila, ibmq-santiago, ibmq-melbourne [3]) are presented. The methods open a possibility to achieve quantum supremacy in detecting energy levels of many-spin systems on many qubit quantum computers.

It is worth noting that graph states are generated by action of the operator of evolution with Hamiltonian of a spin system on a initial state. The states are widely used in quantum information (quantum error correction, quantum cryptography and so on). We consider the graph states of spin systems described by the Ising model. Quantum protocol for quantifying the geometric measure of entanglement of the graph states is proposed. We find the geometric measure of entanglement analytically and with quantum calculations on ibmq-valencia [4, 3]. It is concluded that the geometric measure of entanglement of a spin with other spins in the graph state is related with the degree of the vertex representing the spin in the corresponding graph. Also multi-qubit graph states generated by action of controlled phase shift operators are considered and the geometric measure of entanglement of the states is detected on the quantum device ibmq-athens [5, 3].

Keywords and phrases: quantum graph states, geometric measure of entanglement, energy levels, quantum programming

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Bounded biharmonic mappings

Abstract. We discuss the class of the complex-valued bounded biharmonic mappings. In particular, we present the Schwarz type inequalities for subclasses of this class.

Keywords and phrases: biharmonic mappings, harmonic mappings, Schwartz lemma

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Monogenic functions with values in commutative complex algebras of the second rank with unity and generalized biharmonic equation with simple characteristics

Abstract. Among all two-dimensional algebras of the second rank with unity e over the field of complex numbers $\mathbb{C}$, we found a semi-simple algebra $\mathbb{B}_0 := \{c_1 e + c_2 \omega : c_k \in \mathbb{C}, k = 1, 2\}$, $\omega^2 = e$, containing bases $\{e_1, e_2\}$, such that $\mathbb{B}_0$-valued "analytic" functions $\Phi(xe_1 + ye_2)$ (x, y are real variables) satisfy the fourth order homogeneous partial differential equation of the form:

$$\left(b_1 \frac{\partial^4}{\partial y^4} + b_2 \frac{\partial^4}{\partial x \partial y^3} + b_3 \frac{\partial^4}{\partial x^2 \partial y^2} + b_4 \frac{\partial^4}{\partial x^3 \partial y} + b_5 \frac{\partial^4}{\partial x^4}\right) u(x, y) = 0, \tag{1}$$

where complex coefficients $b_k \in \mathbb{C}$, $k = \overline{1, 5}$, $b_5 \neq 0$, such than the Eq. of characteristics

$$l(s) := b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0, s \in \mathbb{C}, \tag{2}$$

has four pairwise different roots (each root is a simple root).

A set of pairs $(\{e_1, e_2\}, \Phi)$ is described in the explicit form.

Particular cases (under coefficients of Eq. (1)) are considered in [1-3]. A case of Eq. (1) with simple and only nonzero simple characteristics is considered in [4].

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Data stream mining algorithms – mathematical perspective

Abstract. Data stream mining become a very challenging and important field of computer science research [1], mainly due to the enormous growth of data amounts produced in various areas of human activities. Appropriate algorithms are particularly important in cases when the available resources, such as computational power or memory, are not enough to process whole datasets. Traditional data mining or machine learning algorithms are not suitable in this scenario, therefore new methods are required that allow processing data on-the-fly. For example, a common assumption is made that the algorithm can process any data element at most once.

Apart from achieving satisfactory performance from the practical point of view, it is also very important to make a deep insight into understanding of mathematics behind data stream mining algorithms. For example, in methods based on decision trees the problem is to estimate the required number of data elements collected in particular tree nodes to make decisions about subsequent splits. Initially, the Hoeffding's inequality was proposed as a tool for solving this problem [2], however, it turned out that it is not feasible for many nonlinear impurity measures used in decision tree induction, like the information entropy or the Gini index. New tools were proposed instead, such as the McDiarmid's inequality [3] or the Gaussian approximation [4].

Another group of methods that can be adopted to data stream mining tasks are probabilistic neural networks or generalized regression neural networks [5]. It is possible to proof their convergence even in time-varying data distributions, if appropriate assumptions regarding the rate of those changes are made. These methods can be used to perform data distribution and regression function estimations in online manner.

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On some finite difference properties of analytic functions

Abstract. Let consider a function realizing homeomorphism of the closed unit disk onto the closure of a simply connected domain in the complex plane bounded by a smooth Jordan curve conformal in the open unit disk.

Relationship between properties of the boundary of such domain and properties of the considered function was investigated in works by numerous authors: O. D. Kellogg, S. E. Warshawski, J. L. Geronimus, S. J. Alper, R. N. Kovalchuk, L. I. Kolesnik, P. M. Tamrazov. Some close problems were investigated by V. A. Danilov, E. P. Dolzenko, S. V. Kolesnikov, E. M. Dynkin, N. A. Shirokov, S. R. Bell and S. G. Krantz, V. V. Andrievskii, V. I. Belyi, B. Oktay, D. M. Israfilov and others. Certain results in terms of moduli of smoothness of different types (uniform curvilinear, arithmetic, local and integral moduli of smoothness of arbitrary order) were received by author.

Let consider the simply connected domains G_1 and G_2 in the complex plane bounded by the smooth Jordan curves Γ_1 and Γ_2. Let $\tau_m(s_m)$ be the angle between the tangent to Γ_m and the positive real axis, $s_m(z)$ be the arc length on Γ_m $(m = 1, 2)$. Let $w = f(z)$ be a homeomorphism of the closure $\overline{G_1}$ of the domain G_1 onto closure $\overline{G_2}$ of the domain G_2, conformal in open domain G_1.

Theorem. Let integral moduli of smoothness of order k $(k \in \mathbb{N})$ for the functions $\tau_1(s_1)$ and $\tau_2(s_2)$ satisfy conditions

$$\widehat{\omega}_k(\tau_m(s_m), \delta) = O(\omega(\delta)) \quad (\delta \to 0) \quad (m = 1, 2),$$

where $\omega(\delta)$ is normal majorant satisfying Dini condition. Then integral modulus of smoothness of the derivative of the function $w = f(z)$ on Γ_1 satisfies the condition

$$\widehat{\omega}_k(f'(z), \delta) = O(\sigma(\delta)) \quad (\delta \to 0)$$

where $\sigma(\delta)$ is some integral majorant.

In partial case when $\widehat{\omega}_k(\tau_1(s_1), \delta)$ and $\widehat{\omega}_k(\tau_2(s_2), \delta)$ satisfy Holder condition with index α, then integral modulus of smoothness $\widehat{\omega}_k(f'(z), \delta)$ of the derivative of the function $f(z)$ on Γ_1 satisfies Holder condition with the same index α.

Certain results were received for the higher order derivatives of considered function.

Keywords and phrases: conformal mapping, modulus of smoothness

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Morse Lemma

Abstract.

Morse Lemma. Let f be a smooth function of C^∞ class in a certain environment of the point $a \in \mathbb{R}^n$, $n > 1$ such that

$$f(a) = 0, \qquad \nabla f(a) = 0 \qquad \text{and} \qquad \det\left[\frac{\partial^2 f}{\partial x_i \partial x_j}(a)\right] \neq 0.$$

Then there are a diffeomorphism $\varphi : (\mathbb{R}^n, a) \to (\mathbb{R}^n, a)$ of C^∞ class and the integer number $0 \leqslant l \leqslant n$ such that

$$f \circ \varphi(x) = -\sum_{i=1}^{l}(x_i - a_i)^2 + \sum_{i=l+1}^{n}(x_i - a_i)^2 \quad \text{in a certain environment of the point } a,$$

where $a = (a_1, \ldots, a_n)$.

Morse Lemma for holomorphic function. Let f be a holomorphic function in a certain environment of the point $\overset{\circ}{z} \in \mathbb{C}^n$, $n \geqslant 1$ such that

$$f(\overset{\circ}{z}) = 0, \qquad \nabla f(\overset{\circ}{z}) = 0 \qquad \text{i} \qquad \det\left[\frac{\partial^2 f}{\partial z_l \partial z_k}(\overset{\circ}{z})\right] \neq 0.$$

Then there is a biholomorphism φ in a certain environment of the point $\overset{\circ}{z}$ such that

$$f \circ \varphi(z) = \sum_{k=1}^{n}(z_k - \overset{\circ}{z}_k)^2 \quad \text{in a certain environment of the point } \overset{\circ}{z}.$$

where $\overset{\circ}{z} = (\overset{\circ}{z}_1, \ldots, \overset{\circ}{z}_n)$.

These two cases of Morse Lemma will be discussed in detail.

Keywords and phrases: biholomorphism, diffeomorphism, holomorphic function, Morse Lemma, non - degenerate peculiar point, smooth function

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On the order of growth of one class of homeomorphisms

Abstract. Let Γ be a family of curves γ in $\mathbb{R}^n$, $n \geqslant 2$. A Borel measurable function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, (abbr. $\rho \in \operatorname{adm}\Gamma$), if $\int\limits_{\gamma} \rho(x)\, ds \geqslant 1$ for any curve $\gamma \in \Gamma$. Let $p \in (1, \infty)$.

The quantity

$$M_p(\Gamma) = \inf_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)$$

is called p*–modulus* of the family Γ.

For arbitrary sets E, F and G of $\mathbb{R}^n$ we denote by $\Delta(E, F, G)$ a set of all continuous curves $\gamma : [a, b] \to \mathbb{R}^n$, that connect E and F in G, i.e., such that $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in G$ for $a < t < b$.

Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, $x_0 \in D$ and $d_0 = \operatorname{dist}(x_0, \partial D)$. Set $\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}$, $S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n : |x - x_0| = r_i\}$, $\quad i = 1, 2$.

Let a function $Q : D \to [0, \infty]$ be Lebesgue measurable. We say that a homeomorphism $f : D \to \mathbb{R}^n$ is ring Q-homeomorphism with respect to p-modulus at $x_0 \in D$, if the relation

$$M_p(\Delta(fS_1, fS_2, fD)) \leqslant \int\limits_{\mathbb{A}} Q(x)\, \eta^p(|x - x_0|)\, dm(x)$$

holds for any ring $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)$, $0 < r_1 < r_2 < d_0$, $d_0 = \operatorname{dist}(x_0, \partial D)$, and for any measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r)\, dr = 1$.

Denote by ω_{n-1} the area of the unit sphere $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$ in $\mathbb{R}^n$ and by $q_{x_0}(r) = \frac{1}{\omega_{n-1} r^{n-1}} \int\limits_{S(x_0, r)} Q(x)\, d\mathcal{A}$ the integral mean over the sphere $S(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| = r\}$, here $d\mathcal{A}$ is the element of the surface area. Let $L(x_0, f, R) = \sup\limits_{|x - x_0| \leqslant R} |f(x) - f(x_0)|$.

Theorem. *Suppose that $f : \mathbb{R}^n \to \mathbb{R}^n$ is a ring Q-homeomorphism with respect to p-modulus at a point x_0 with $p > n$ where x_0 is some point in $\mathbb{R}^n$. Then for all numbers $r_0 > 0$ the estimate*

$$\varliminf_{R \to \infty} \left(L(x_0, f, R) \left(\int\limits_{r_0}^{R} \frac{dt}{t^{\frac{n-1}{p-1}}\, q_{x_0}^{\frac{1}{p-1}}(t)} \right)^{-\frac{p-1}{p-n}} \right) \geqslant \left(\frac{p-n}{p-1} \right)^{\frac{p-1}{p-n}} > 0$$

holds.

Keywords and phrases: Ring Q-homeomorphisms, p-modulus of a family of curves, quasiconformal mappings

Acknowledgements. This work was supported by the budget program "Support of the development of priority trends of scientific researches" (KPKVK 6541230).

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Skyrmions and antiskyrmions in ferrimagnetic multilayers

Abstract. Skyrmions and antiskyrmions are topologically protected spin structures with opposite vorticities [1]. While the former have been extensively researched over the past years in a variety of magnetic materials and systems, the latter were exclusive to materials with D2d-symmetry [2]. These magnetic spin textures are promising in realizing future spintronic devices and data storage technologies. Thus, their coexistence may show fascinating physics and potential for novel spintronic devices.

In a recent publication [3] the coexistence of dipolar stabilized first and second-order antiskyrmions, skyrmions and type-II bubbles in Fe/Gd multilayers was observed. In this conference talk, we present the underlying micromagnetic simulations from [3]. Here, we employ finite elements and differences based micromagnetic simulation codes, to determine the range of material parameters where the spin textures are independently stable, and demonstrate the coexistence of spin textures in realistic system dimensions, see Fig. 1. Furthermore, we qualitatively describe the nucleation process of antiskyrmions when external magnetic fields perpendicular to the film plane are applied. Additionally, we demonstrate with new micromagnetic simulations that the antiskyrmions can also be stabilized when films of different magnetic properties are combined, and thus, lead to an overall reduction of saturation magnetization. This allows us, to extend the range of usable magnetic materials where the skyrmions and antiskyrmions can coexist.

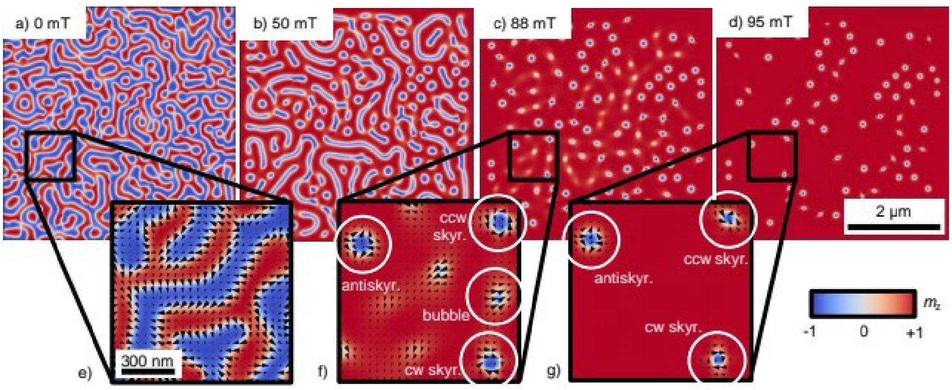

Fig. 1: *Simulated magnetic domain morphology for different applied magnetic fields, where the coexistince of skrymions, antiskyrmions and type-II bubbles are observed [3].*

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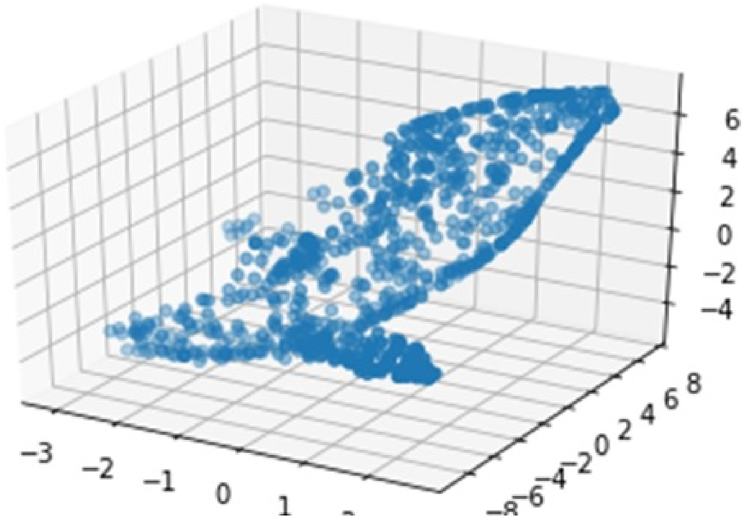
The application of unsupervised learning to the AC susceptibility measurements of High-Temperature Superconductors

Abstract. Machine learning (ML) is the study of computer algorithms that improve automatically through experience. ML algorithms are built on a mathematical model based on data, in order to make predictions or decisions without being explicitly programmed to do so. Unsupervised learning (UL) is a subfield of ML. UL algorithms look for previously undetected patterns in a dataset with no pre-existing labels and with a minimum of human supervision. Great progress has been made in a quest to discover, develop or refine various machine learning algorithms in recent years and new ways of data analysis have been shown. The ML application to the analysis of datasets is a state of the art technique which allows to make breakthroughs in various areas of science and engineering.

Our work aims to provide a first insight into application of clustering techniques to the large dataset of AC susceptibility measurements of High-Temperature Superconductors (HTS). It should allow recovering known relationships between different types of HTS and their superconducting properties.

We show that it is possible to represent the most significant features of a single AC measurement of a HTS sample as 5 numerical values by using a Convolutional 1D Autoencoder and the Bag Of Words model. The most distant 5D representations of $\chi(T)$ are for samples, which have the most different superconducting properties i.e. thin layer HTS and grinded and pressed polycrystalline HTS so the 5D representation of the $\chi(T)$ dataset preserves the most important features of the measurement of the HTS sample. However the cluster analysis of the 5D $\chi(T)$ dataset by two clustering algorithms did not reveal the existence of clearly distinct classes of $\chi(T)$ measurements. Though a t-SNE visualisation (fig. 1) in 3D space shows that some clustering exists and part of the measurements are mainly arranged on some sort of cluster boundary. Therefore, more advanced analysis could be performed.



Acknowledgements: This work was supported by The National Science Center Poland under Project No. DEC-2018/02/X/ST3/01741.

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Comparative study of magnetic properties of Mn^{3+} magnetic clusters in GaN using classical and quantum mechanical approach

Abstract. Currently, simulations of many-body quantum systems are known to be computationally too demanding to be solved on classical computers. The main problem is that the computation time (number of elementary operations) and memory necessary for performing the calculations usually grow exponentially with the number of particles N. An efficient approach to simulate many-body quantum systems is the use of classical approximation. For example, recently, it has been shown that magnetic properties of ferromagnetic (Ga,Mn)N at $T = 2$ K can be successfully described using atomistic spin model and the Landau-Lifshitz-Gilbert (LLG) approach, applied for an ensemble of about 10000 Mn spins [1]. However, it is known that, at low temperatures the allowed spin fluctuations in this approach are overestimated what corresponds to enhanced thermal fluctuations [2]. It is important then to somehow asses the validity of this classical approximation. For practical reasons the quantum simulations of interacting Mn^{3+} ions in GaN, coupled by ferromagnetic super-exchange interaction $-J\mathbf{S}_1 \cdot \mathbf{S}_2$, are restricted up to small magnetic clusters. Therefore in this work, we compare the results of numerical calculations of magnetic clusters (singlet, pairs,triplets and quartets) in (Ga,Mn)N where the Mn spins are treated classically with those where they are treated quantum-mechanically (crystal field model) [3, 4]. In the first case, we solve the LLG equation, that describe the precessional dynamics of spins represented by classical vectors. On the other hand, in crystal field model, the Mn^{3+} state (d^4 configuration, with $S = 2, L = 2$) is characterized by the set of orbital and spin quantum numbers $|m_s, m_L >$. The relevant energy level structure of singlet, pair, triplet and quartet of Mn ions are found by the numerical diagonalization of full 25x25, $25^2 \times 25^2$, $25^3 \times 25^3$ and $25^4 \times 25^4$ Hamiltonian matrix respectively. Particular attention is paid to use numerical parameters that ensure the same single ion magnetic anisotropy in classical and quantum approximation. Finally, a detailed comparative study of magnetization $\mathbf{M}(\mathbf{H}, T)$ as a function of magnetic field $\mathbf{H}$, temperature T, number of ions in given cluster and the strength of super-exchange interaction J, obtained from both approaches will be presented.

Keywords and phrases: magnetic clusters, crystal field model, atomic spin model.

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Clusterization in action

Abstract. I will describe a pipeline for unsupervised machine learning/clusterization that proved to be extremely useful in applications. I will present its application to problems from physics/chemistry and management sciences. The talk will be accessible to mathematicians, physicists, and computer scientists.

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Cyclostationary Time Series. Functional Data Analysis and Time Series.

Abstract. Cyclostationary time series are very popular in various applications. In recent 5 years, over 2500 publications have appeared where the concept, theory and applications of cyclostationarity is studied. In my talk, I would like to give a short overview of fundamental inferential problems related to cyclostationary models. Moreover, in view of increasing popularity of functional data analysis in big data context, I would like to investigate functional versions of cyclostationary models and their properties.

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Past and present extremal problems for series of holomorphic functions of several variables

Abstract. The lecture is devoted to recalling some results related to estimates for coeffcients of holomorphic functions on bounded complete n-circular domains of $\mathbb{C}^n$. During the talk we will present, in particular, various forms of the aforementioned estimates, old and current. Also we will inform about their sharpness, efficiency and uniqueness of the extremal functions. We will focus mainly on the results in families composed of functions, which are bounded, having a positive real part and functions defined by the well-known planar concept of starlikeness and convexity. We will based on the papers:

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Hyperbolic curves and integration of bicomplex functions

Abstract. The notion of hyperbolic straight lines and hyperbolic curves in the set $\mathbb{BC}$ of bicomplex numbers, or in the n dimensional hyperbolic module $\mathbb{D}^n$ have been helpful for a better understanding of some analytical facts in both theories. In this talk we will analyze some of these facts. For example, how can be described the bicomplex stereographic projection or, as the title of the talk says, how the hyperbolic–curves help to establish some of the integral theorems in the bicomplex context.

Keywords and phrases: Bicomplex numbers, hyperbolic numbers, hyperbolic curves, partial order in the set of hyperbolic numbers

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Guthrie and Nymann Cantorval

Abstract. We present the selected problems connected with Guthrie and Nymann Cantorval, that is

$$GN = A(x_n) \quad \text{for } x_{2n-1} = \frac{3}{4^n},\, x_{2n} = \frac{2}{4^n},$$

where $A(x_n) = \left\{\sum \varepsilon_n x_n : (\varepsilon_n) \in \{0,1\}^\omega\right\}$ is the achievement set of the sequence (x_n). The talk will include the counterexample for Kakeya hypothesis, recovering techniques for the sequence from its achievement set which are based on the center of distances, some results in cardinal functions theory and connections with iterated function systems.

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About the class of completely integrable evolutionary models

Abstract. A one class of the completely integrable evolutionary models is considered. Both known and previously unknown analytical solutions of evolutionary equations, expressed in terms of the Euler beta function, are presented. It is shown that, in some cases, it is possible to obtain an analytical solution for the harvesting evolutionary equations. The using of a non-power-law generating function allows us to propose previously unused evolutionary models which are also completely integrable. The obtained results can be applied in ecology, physics, chemistry and medicine.

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CMTJ – Simulation tool for multilayer spintronics devices

Abstract. We present CMTJ - a fast macrospin library that allows for simulating ferromagnetic layers, multilayer structures, and systems of multilayer structures. The package is exposed both in C++ and Python languages with an easy interface for defining custom simulations. In its core, CMTJ is a macromagnetic model that solves the Landau-Lifshitz-Gilbert-Slonczewski (LLGS) equation taking into account a variety of effective field contributions such as layer couplings, anisotropy, demagnetisation and dipole interactions. The simulation may be interpreted both in STT (spin-transfer)[1] and SOT (spin-orbit-torque) [2] sense, which is supported by exposing adequate interfaces for experimental verification or theoretical exploration. In addition, we implement a simple model for coupling multiple layers in parallel or series [3] connection. Finally, CMTJ employs a stochastic differential equation (SDE) solver allowing for simulations with temperature, which acts as a stochastic proxy. We justify the accuracy of the CMTJ by displaying a range of results that agree with modern experimental measurements and techniques, such as: spin-diode effect, ferromagnetic curves, pulse-induced magnetometry, magnetisation field loops, resistance-field loop and harmonic detection. The simulation package also comes with a GUI named PyMag that entails part of the CMTJ's capability for easier verification of the experimental results against the simulated data. Repository with the package is open-source and available here: https://github.com/LemurPwned/cmtj.

Keywords and phrases: magnetics, spintronics, numerical simulation, MTJs

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On lineal convexity generalized to commutative and non-commutative algebras

Abstract. The notion of lineally convex domains in the finite-dimensional complex space $\mathbb{C}^n$ [1], [2] and some of their properties are generalized to the finite-dimensional space $\mathcal{A}^n$, $n \geqslant 2$, that is the Cartesian product of n commutative or non-commutative, associative algebras $\mathcal{A}$. Namely, a domain in $\mathcal{A}^n$ is said to be ***(locally) $\mathcal{A}$-lineally convex*** if for any boundary point $\boldsymbol{w}$ of the domain there exists a hyperplane in $\mathcal{A}^n$ passing through $\boldsymbol{w}$ but not intersecting the domain (in some neighborhood of $\boldsymbol{w}$).

There are obtained the separate necessary and sufficient conditions of the local $\mathcal{A}$-lineal convexity of domains with smooth boundary, where $\mathcal{A}$ is commutative [5], [6]. The conditions are obtained in terms of nonnegativity and positivity of the differential of the second order of a real function defining the domain, respectively. Moreover, the sign of the differential is determined on the boundary of the domain and on the vectors of the hyperplane tangent to the domain. These conditions are a generalization of the well-known conditions of the local linear convexity of a domain with smooth boundary in $\mathbb{C}^n$ obtained by B. Zinoviev [3]. Similar conditions where also obtained for the non-commutative Clifford algebras and the generalized quaternions algebras [4].

Using the algebra of dual numbers as an example, the existence of the spaces $\mathcal{A}^n$ and domains with smooth boundary in them with several distinct tangent hyperplanes at the same point of the domain boundary is shown [6]. There are also given the example of the three-dimensional algebra $\mathcal{A}_1$ such that a domain in $\mathcal{A}_1^n$ has infinitely many tangent hyperplanes at some its boundary point and the example of the three-dimensional algebra $\mathcal{A}_2$ such that any domain in $\mathcal{A}_2^n$ with boundary of class C^2 has the unique tangent hyperplane at any boundary point, similar to the case of the algebra $\mathbb{C}$ [6].

Keywords and phrases: lineally convex set, commutative algebra, Clifford algebras, quaternions, linear form, quadratic form, differential forms, formal derivatives.

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On the Koebe Quarter Theorem for certain polynomials

Abstract. The presented results concern close-to-convex polynomials with all zeros of derivative in the unit circle $\mathbb{T}$. The minimal disc containing all images of the unit disc $\mathbb{D}$ and the maximal disc contained in all images of $\mathbb{D}$ through polynomials of degree 3 and 4 were determined in [4]. Moreover, all extremal functions for both problems were received.

Joint work with: Szymon Ignaciuk

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Linear combinations of projections in von Neumann factors

Abstract. Von Neumann algebra $\mathcal{M}$ is a natural generalization of the space of all operators in a Hilbert space, crucial in quantum physics. Let $\mathcal{M}$ be a von Neumann algebra and $\mathcal{M}_h$ its self-adjoint part. We shell discuss the following questions.

1. Is each operator $a \in \mathcal{M}_h$ (finite, real) linear combination of projections from $\mathcal{M}$?
2. If the answer is yes, is there a number n such that each (self-adjoint) operator can be written as a linear combination of n projections?
3. If the answer to the second question is yes, then what is the smallest n?

The answers depend on the type of the algebra. We shall give fairly precise answers for factors.

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Charge order and Strange metals in cuprate superconductors

Abstract. Charge orders and charge fluctuations have been ubiquitously observed in the phase diagram of Cuprate superconductors. We will review the experimental status of these various observations, differentiating the under-doped region and the optimally-doped and over-doped ones. Various theories have been advanced to explain the presence of these orders and their implication for our understanding of the pseudo-gap, from the idea of "vestigial order" to the one of "fluctuating Pair Density Wave (PDW)". We will discuss these theoretical approaches in direct comparison with experiments. We will then introduce a proposal of "fractionalization of a PDW" in order to explain the pseudo-gap state. We will show that this idea produces a strong phenomenology, especially ARPES experiments, and giving a clue for the puzzling transport properties recently reported in the optimally doped and over-doped regions. We will then focus on the strange metal phase of those compounds and make a proposal for electric transport in this phase.

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Monogenic functions in a harmonic algebra

Abstract. An associative commutative three-dimensional algebra $\mathbb{A}_3$ with unit 1 is *harmonic* if in $\mathbb{A}_3$ there exists a *harmonic* basis $\{e_1, e_2, e_3\}$ satisfying the following equality

$$e_1^2 + e_2^2 + e_3^2 = 0$$

provided that $e_j^2 \neq 0$ for $j = 1, 2, 3$.

It is well-known that the harmonic algebra $\mathbb{A}_3$ exists only over the complex field $\mathbb{C}$. Moreover, there are three harmonic three-dimensional algebras exactly, and all harmonic bases in these algebras are constructed (see [1]).

We consider a harmonic algebra $\mathbb{A}_3$ with the basis $\{1, \rho, \rho^2\}$ such that $\rho^3 = 0$. There are harmonic bases in $\mathbb{A}_3$, in particular, $e_1 = 1$, $e_2 = i + \rho^2$, $e_3 = (1 - i)\rho$.

Let $E_3 := \{\zeta := xe_1 + ye_2 + ze_3 \, : \, x, y, z \in \mathbb{R}\}$ be the linear span generated by the vectors e_1, e_2, e_3 over the real field $\mathbb{R}$.

We say that a function $\Phi\colon \Omega \longrightarrow \mathbb{A}$ is called *differentiable in the sense of Gâteaux* in a domain $\Omega \subset E_3$ if for every $\zeta \in \Omega$ there exists an element $\Phi'_G(\zeta) \in \mathbb{A}$ such that

$$\lim_{\delta \to 0+0} \left(\Phi(\zeta + \delta h) - \Phi(\zeta)\right) \delta^{-1} = h\Phi'_G(\zeta) \quad \forall h \in E_3.$$

Obviously, the *Gâteaux derivative* $\Phi'_G(\zeta)$ is a function of the variable ζ and is a generalization of the classical directional derivative.

A function $\Phi\colon \Omega \longrightarrow \mathbb{A}_3$ is called *monogenic* in a domain $\Omega \subset E_3$, if Φ is continuous and has Gâteaux derivative $\Phi'_G(\zeta)$ at every point $\zeta \in \Omega$ (see [2, 3]).

For monogenic functions $\Phi(\zeta)$ taking values in the algebra $\mathbb{A}_3$, we proved analogs of Cauchy integral theorem and Cauchy integral formula that yields the Taylor expansion of monogenic function in the usual way. An analog of Morera theorem is also proved in the usual way. Thus, as in the complex plane, one can give different equivalent definitions of monogenic functions $\Phi(\zeta)$ taking values in the algebra $\mathbb{A}_3$ (see [2]).

We consider a problem of weakening the conditions of monogenity for functions that take values in the algebra $\mathbb{A}_3$. We proved an analog of Menchoff–Trokhymchuk theorem that takes into account the algebraic structure of algebra (see [4]).

Keywords and phrases: harmonic algebra, monogenic function, Cauchy integral theorem, Cauchy integral formula, Taylor expansion, Morera theorem, Menchoff–Trokhymchuk theorem

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Towards analog electronic simulation of electrostatically interacting quantum particles

Abstract. Despite the technological advances and ever growing computational power, digital computers still struggle with the advanced integration and differentiation. Quantum mechanics depends on the Schrodinger equation which necessities solving the advanced differential equations. On the other hand, the analog computers can easily integrate and differentiate using the physical relations inherent in circuits.

The goal of this project is to demonstrate the potential of the analog computers in simulation of the quantum phenomena, by designing the circuit capable of solving the set of the equations describing the interaction of two electrostatically charged particles in nanowires. This project could be developed into the wider concept of modeling the quantum mechanics using the analog electronics.

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Fundamental description of Wannier qubits of any topology in semiconductor by analytical and numerical computations

Abstract. Justification of tight-binding model from Schroedinger formalism for various topologies of position-based semiconductor qubits is presented in this work. Simplistic tight-binding model allows for description of single-electron devices at large integration scale. However it is due to the fact that tight-binding model omits the integro-differential equations that arise from electron-electron interaction in Schroedinger model. Two approaches are given in derivation of tight-binding model from Schroedinger equation. First approach is conducted by usage of Green functions obtained from Schroedinger equation. Second approach is given by usage of Taylor expansion applied to Schroedinger equation. The obtained results can be extended for the case of many Wannier qubits with more than one electron and can be applied to 2 and 3 dimensional model. Furthermore various correlation functions are proposed in Schroedinger formalism that can account for static and time-dependent electric and magnetic field polarizing given Wannier qubit system. One of the central results of presented work relies on the emergence of dissipation processes during smooth bending of semiconductor nanowires both in the case of classical and quantum picture. Presented results give the base for physical description of electrostatic Q-Swap gate of any topology using open loop nanowires. We observe strong localization of wavepacket due to nanowire bending.

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Bicomplex Bergman Projection

Abstract. The theory of Bergman spaces is a classic topic in the Complex Analysis; on the other hand, in recent years the theory of bicomplex holomorphic functions has consolidated its development, (see [4], [1], [2]). Bicomplex holomorphic theory shows that it is quite adequate to deal with some analogous of the classical holomorphic functions spaces on the unit complex disk, but now defined in the bidisk. Precisely, we introduce the bicomplex weighted Bergman spaces $\mathbb{BC}\mathcal{A}^p_\alpha$ on the bidisk $\mathbb{U} \subset \mathbb{C}^2$ and its associated bicomplex weighted Bergman projection $\mathbf{P}_{k,\alpha}$. Previous works in this direction appear in [3] and [5]. The bicomplex Bloch space is defined in the bidisk $\mathbb{U}$ and we study the bicomplex Bergman projection onto the bicomplex Bloch space.

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Quaternion Quantum Mechanics: from Hamilton spacetime to the displacement four-potential in the Planck-Kleinert crystal

Abstract. We show the well-founded mathematical roots of the quaternion quantum mechanics (QQM) derived from the Cauchy model of the elastic continuum. The Cauchy classical balance equations for the isotropic ideal elastic material and the Hamilton quaternion algebra $\mathbb{Q}$, allow a rigorous derivation of the quaternion form of the non- and relativistic wave equations. The family of the wave equations and the Poisson equation are a straightforward consequence of the quaternion representation of the Cauchy theory

$$\begin{cases} \frac{1}{c^2}\frac{\partial^2\sigma}{\partial t^2} - (n+1)\triangle\sigma\cdot\sigma^* = 0, \\ (n-2)\triangle\sigma_0 + n\triangle\hat{\phi} - k_n^2\sigma\cdot\sigma^* = 0, \end{cases} \qquad for \;\; n = 0, 1, 2, \ldots,$$

where $\sigma = \sigma + \hat{\phi} \in \mathbb{Q}$ and $\sigma^* = \sigma_0 - \hat{\phi} \in \mathbb{Q}$.

The QQM can be regarded as representing the physical reality of the elastic continuum. The problem of the Schrödinger equation, where imaginary 'i' should emerge, is solved. This interpretation is a fundamental for the ontology of quantum mechanics, and demonstrates that, besides the Bohmian mechanics, the complete ontological interpretations of quantum theory exists. We will show the simulation of two particles in $\mathbb{R}^3$.

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Topological Evolution of Magnetically Knotted-Linked Physical Systems

Abstract. Variety of plasma phenomena in the laboratory, outer space and astrophysical environments indicate formation of unusual geometrical structures in the electromagnetic (em) fields, facilitating adaptation of modern topological methods in the description of these configurations. Of particular interest is the evolution of physical system through subsequent topological reconfigurations as a reference to experimental data. These robust processes will be (1) surveyed for various free-space and magnetohydrodynamics (MHD) structures resulting in fundamental physical implications to nuclear fusion, astrophysics and beyond, and (2) applied to topological analysis of magnetized structures observed by a satellite which is reaching the Sun vicinity and at similar astrophysical features.

Although the common approach to Maxwell equations involves expansion in partial eigenwaves with propagation characteristic determined by the ambient medium, there exists a less known solution based on Hopf map relating each point in S2 to a circle (fiber) in S3 with all fibers linked to each other and lie on nested toroidal surfaces. The resulting configuration in empty space forms divergence free magnetic/electric vectors where the linear superposition is replaced by congruence, which assures the invariance of the Pointing flux (1), allowing information encoding in the energy flow with important implications for quantum computing. Similar Hopf-like self-stable toroidal structures emerge from resistive MHD reconnection of helical magnetic field in force equilibrium (2). They fall into the category of magnetic knots which are the subject of mathematical research.

One of the surprising observations by Parker Solar Probe satellite in solar vicinity were ubiquitous chains of large reversals in the radial component of the magnetic field. The proposed scenario of these “switchback” encounters is inherently related to the topological features of magnetic braids and knots (3): (a) emergence of braided coronal fields based on data and Artin topological braid theory and Markov moves, (b) novel braids’ reconfiguration path morphing them into magnetic knots (Alexander theorem) whose stability is boosted by their topological invariants, (c) incorporation of the knots into the expanding solar wind and (d) intermittent recombination of knots through Knot Sum into elongated chain of field inversions, resulting in magnetic switchbacks. Similar structures are proposed for the description of Herbig-Haro outflows as observed during planet formation and to laboratory experiments of high-energy density jets driven by toroidal magnetic field. We conjecture that a huge number of astrophysical bodies become sources of localized quasi-stable helical magnetic structures.

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Approximation of some class of holomorphic functions by Cesàro means

Abstract. Let f be a function holomorphic in the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and let

$$f(z) = \sum_{j=0}^{\infty} \widehat{f}_j z^j, \quad \widehat{f}_j := \frac{f^{(j)}(0)}{j!}$$

be its Taylor expansion.

The Cesàro means $\sigma_n^\alpha(f)$, $n = 1, 2, \ldots$, $\alpha > -1$, for f are defined by

$$\sigma_n^\alpha(f)(z) = \frac{1}{A_{n-1}^\alpha} \sum_{j=0}^{n-1} A_{n-j-1}^\alpha \widehat{f}_j z^j, \quad A_k^\alpha := \frac{\Gamma(k+\alpha+1)}{\Gamma(k+1)\Gamma(\alpha+1)} = \binom{k+\alpha}{k},$$

where $\Gamma(n)$ is Gamma function.

Let B^1 be a class of holomorphic functions f with $\sup\limits_{z\in\mathbb{D}} |f'(z)| \leq 1$. Then for all $n \in \mathbb{N}$ and $z \in \overline{\mathbb{D}}$ the equality hold

$$\max\left\{ \sum_{k=0}^{n-1} (A_k^\alpha)^2 \left| f(z) - \sigma_{k+1}^\alpha(f)(z) \right|^2 : f \in B^1 \right\} = |z|^2 \sum_{k=0}^{n-1} \left(A_k^{\alpha-1} \right)^2.$$

If $z \neq 0$ the maximum is attained for the function $f^*(z) = az + b$, a, $b \in \mathbb{C}$, $|a| = 1$.

Keywords and phrases: Holomorphic functions, Cesàro means, exact estimate.

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Approximation of classes of harmonic bounded functions by Fejer means

Abstract. Let hB^1 be a class of harmonic in disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ functions, such that $\|f'\|_\infty < 1$. The Fejer means $\sigma_n[f]$ for harmonic function f are defined by

$$\sigma_n[f](z) = \sum_{|k|\le n-1} \left(1 - \frac{|k|}{n}\right) \widehat{f}_k \epsilon_k(z), \quad \widehat{f}_k = \frac{f^{(k)}(0)}{k!}, \quad \epsilon_k(z) = \begin{cases} \overline{z}^{|k|}, & k \le -1, \\ z^{|k|}, & k > 0. \end{cases}$$

Every function $f \in C^q_\beta := hB^1|_{|z|=q}$ given by the Poisson integral

$$f(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(x+t) P^q_\beta(t)\, dt, \quad P^q_\beta(t) = \sum_{k=1}^{\infty} q^k \cos kt,$$

where $\varphi \in W^1$. In our research we study the asymptotic behavior of upper bounds of deviations of Fejer means on some classes C^q_β. Particularly, we obtain the following statement. Let $0 \le c \le \frac{\pi}{2}$ and $\int_{-\pi}^{\pi} \varphi(t)\, dt = 4c$, then the equality hold as $n \to \infty$

$$\sup_{f \in C^q_\beta} \|f - \sigma_n[f]\| = \frac{q}{\pi n} \frac{4\cos c}{1 - 2q \sin c + q^2} + O(1) \frac{q^n}{n(1-q)^3}, \quad q \le 2 - \sqrt{3}.$$

Keywords and phrases: harmonic function, Fejer means, Poisson integral

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Asymptotically best possible Lebesgue-type inequalities for the Fourier sums on the sets $C^{\alpha,r}_{\beta}L_p$

Abstract. For the functions f, which can be represented in the form of the convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int_{-\pi}^{\pi}\sum_{k=1}^{\infty} e^{-\alpha k^r}\cos(kt - \frac{\beta\pi}{2})\varphi(x-t)dt, \varphi \in L_p, \varphi \perp 1, \alpha > 0,\ r \in (0,1), a_0, \beta \in \mathbb{R},$$

we establish Lebesgue-type inequalities.

In these inequalities uniform norms of deviations of Fourier sums $\|f - S_{n-1}\|_C$ are expressed via best approximations $E_n(\varphi)_{L_p}$ of functions φ by trigonometric polynomials in the metric of space L_p. We show that obtained estimates are asymptotically best possible.

Keywords and phrases: Lebesgue-type inequalities, Fourier sums, generalized Poisson integrals

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Hypercomplex method for solving linear PDEs

Abstract. Algebraic-analytic approach to constructing solutions for given linear partial differential equations were investigated in many papers. It involves solving two problems. Problem (P 1) is to describe all the sets of vectors $e_1, e_2, \ldots, e_d$ of commutative associative algebra, which satisfy the polynomial characteristic equation (or specify the procedure by which they can be found). And the Problem (P 2) is to describe all the components of monogenic (i. e., continuous and differentiable in sense of Gateaux) functions. Earlier we got a constructive description of all monogenic functions with values is obtained in an arbitrary finite-dimensional commutative associative algebra over the field $\mathbb{C}$. Further we states that it is enough to limit the study of monogenic functions in algebras with the basis of $\{1, \eta_1, \eta_2, \ldots, \eta_{n-1}\}$, where $\eta_1, \eta_2, \ldots, \eta_{n-1}$ are nilpotents. In addition, it is showed that in each algebra with a basis of the form $\{1, \eta_1, \eta_2, \ldots, \eta_{n-1}\}$ the polynomial characteristic equation has solutions. That is, the problems (P 1) and (P 2) are completely solved on the classes of commutative associative algebras with the basis $\{1, \eta_1, \eta_2, \ldots, \eta_{n-1}\}$. It is worth noting that in a finite-dimensional algebra a decomposition of monogenic functions has a finite number of components, and therefore, it generates a finite number of solutions of a given partial differential equations. In this report we propose a procedure for constructing an infinite number of families of solutions of given linear differential equations with partial derivatives with constant coefficients. We use monogenic functions that are defined on some sequences of commutative associative algebras over the field of complex numbers. To achieve this goal, we first study the solutions of the so-called characteristic equation on a given sequence of algebras. Further, we investigate monogenic functions on the sequence of algebras and study their relation with solutions of partial deferential equations. The proposed method is used to construct solutions of some equations of mathematical physics. In particular, for the three-dimensional Laplace equation and the wave equation, for the equation of transverse oscillations of the elastic rod and the conjugate equation, a generalized biharmonic equation and the two-dimensional Helmholtz equation. We note that this method yields all analytic solutions of the two-dimensional Laplace equation and the two-dimensional biharmonic equation (Goursat formula).

Keywords and phrases: commutative associative algebra, monogenic function, characteristic equation, expansion of commutative algebra, PDE, infinite-dimensional family of solutions

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Methods for Parameters Optimization in SVMs and SVRs

Abstract. Support Vectors Machines represents a ML techniques that proof good results in solving classification (SVMs) and regression (SVRs) problems form many fields. To handle the non-linearity of data, the kernel trick is applied. The data are mapped into a space of higher dimension where they are linearly separable. This kernels used in these transformations have to satisfy the Mercer theorem. One of the drawback of the kernel trick is that it introduces many parameters. A good choice of the kernel and of its parameters is essential for the model performances. In this paper we present a method for parameters optimization in SVM and SVR models using metaheuristcs inspired from nature. A sensitivity study and validation of the method are presented in the end.

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Modeling of electrostatic programmable quantum gates of different topologies

Abstract. Single electron devices implemented in the chain of coupled semiconductor quantum dots becomes quite promising way of implementation of qubits, quantum logical gates and quantum communication systems due to usage of well-developed CMOS technology that guarantees very high integration. The concept of programmable quantum matter and thus quantum circuits can be modeled by usage of quasi one-dimensional models of semiconductor nanowires in Schrödinger or tight-binding formalisms. In such case, open loop nanowires of arbitrary topology can be approximated by quasi one-dimensional description. The presented scheme can be easily generalized to N interacting electrons placed at N different semiconductor nanowires, whose functionality can be regulated with proper external biasing electric and electromagnetic fields. In such way, quantum information processing can be studied in dependence on different topologies of semiconductor nanowires in various electromagnetic conditions.

Keywords and phrases: quantum computers, quantum gates

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Local Properties of Solutions of Nonlinear Beltrami Equation

Abstract. Let G be a domain in a complex plane $\mathbb{C}$, so a connected and open subset $\mathbb{C}$, and let $\mu\colon G \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. (almost everywhere) in G. *Beltrami equation* is called an equation of the form

$$f_{\overline{z}} = \mu(z) f_z\,, \tag{3}$$

where $f_{\overline{z}} = \frac{1}{2}(f_x + i f_y)$, $f_z = \frac{1}{2}(f_x - i f_y)$, $z = x + iy$, f_x and f_y are partial derivatives of the mapping f of x and y, respectively.

Let $\sigma\colon G \to \mathbb{C}$ be a measurable function and $m \geqslant 0$. Consider in the polar coordinate system (r, θ) the following equation:

$$f_r = \sigma(re^{i\theta})\,|f_\theta|^m\, f_\theta, \tag{4}$$

where f_r and f_θ are partial derivatives of the mapping f of r and θ, respectively. Considering the formulas $r f_r = z f_z + \overline{z} f_{\overline{z}}$, $f_\theta = i(z f_z - \overline{z} f_{\overline{z}})$, the equation (4) may be written in a complex form:

$$f_{\overline{z}} = \frac{z}{\overline{z}}\,\frac{\sigma(z)\,|z|\,i|z f_z - \overline{z} f_{\overline{z}}|^m - 1}{\sigma(z)\,|z|\,i|z f_z - \overline{z} f_{\overline{z}}|^m + 1}\, f_z. \tag{5}$$

Note that when $m = 0$ the equation (5) comes to the usual Beltrami equation (3) with a complex coefficient

$$\mu(z) = \frac{z}{\overline{z}}\,\frac{\sigma(z)\,|z|\,i - 1}{\sigma(z)\,|z|\,i + 1}\,.$$

If put $m = 0$ and $\sigma = -i/|z|$ into (5) then we come to the famous Cauchy-Riemann system. We will continue to assume that $m > 0$.

The mapping $f\colon G \to \mathbb{C}$ is called *regular at the point* $z_0 \in G$, if f has a complete differential and its Jacobian $J_f = |f_z|^2 - |f_{\overline{z}}|^2 \neq 0$ at this point. The homeomorphism f of the Sobolev class $W^{1,1}_{\mathrm{loc}}$ is called *regular* if $J_f > 0$ a.e. *A regular homeomorphic solution* of the equation (5) is called regular homeomorphism $f\colon G \to \mathbb{C}$, that satisfies the equation (5) a.e. in G.

Theorem. *Let $f\colon \mathbb{B} \to \mathbb{C}$ be a regular homeomorphic solution of the equation* (5) *of the Sobolev class $W^{1,2}_{\mathrm{loc}}$ with the normalization $f(0) = 0$. Then for some number $\varepsilon_0 \in (0,1)$ the following condition is fulfilled*

$$\liminf_{z\to 0} |f(z)| \left(\int\limits_{|z|}^{\varepsilon_0} \frac{dt}{I_{m,\sigma}(t)} \right)^{\frac{1}{m}} \leqslant c_0 < \infty,$$

where $I_{m,\sigma}(t) = \left(\int\limits_{\gamma_t} \frac{ds}{|z|(\operatorname{Im}\overline{\sigma}(z))^{\frac{1}{m+1}}} \right)^{m+1}$ *and* $c_0 = (2\pi)^{-\frac{m+1}{m}} m^{-\frac{1}{m}}$.

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On some exact solutions of certain nonlinear PDE's in physics

Abstract. In this presentation some exact solutions of equations of baby Skyrme model (investigated among others in [1]) and certain heavenly-family equations (some of such equations were investigated among others in [11] and [13]), are presented. Two methods have been applied for obtaining these solutions: strong necessary conditions (for the first time presented in [6], developed in [7], [8], and recently developed also in [2]), and structural decomposition method (for the first time presented in [10]). Strong necessary conditions can be applied to obtain among others, Bogomolny equations (firstly obtained for some field-theoretical models, in [3], [4] and [5], by completing to a square), and this aspect was considered in [6], [9], and recently also in [2]. Such equations were derived for among others, baby Skyrme model [12]. These equations are investigated during this talk.

Keywords and phrases: soliton, baby Skyrme model, asymmetric heavenly equation, mixed heavenly equation

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Description of coupled field induced Josephson junctions in curved Ginzburg-Landau equation

Abstract. Ginzburg-Landau equation is derived in context of curved nanocables in quasi-one dimensional approximation. This equation is incorporated into the concept of coupled robust field induced Josephson junctions placed in complex electromagnetic environments. Such structures are made by polarization of superconducting nanostructures by magnetic fields. The methodology of modeling of structures and possible implementations is introduced. The presented scheme is expected to implement and describe both classical and quantum computers with use of Josephson junction and artificial evolution. In such cases one can obtain unexpected new topologies of circuits that can contribute in enhancement of known and used circuit schemes. Furthermore one can obtain the non-invasive detectors of moving charged particles from robust field induced Josephson junctions which are illustrated by examples. The analogy of description electrostatic quantum gates build from semiconductor of any open loop shape and field-induced Josephson junctions using semiconductors of any shape are given.

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Non-commutative Galois theory and its application to general evolution

Abstract. In this talk, we will discuss evolutions and hyper complex analysis in terms of non-commutative Galois theory in the following 4 parts:

(Part 1) We can find solutions of an algebraic equation of degree n with rational number coefficients in the following successive extensions: (1) In the case $n = 2$, we find solutions in the binary extension: $\sqrt[2]{\ }$ $(= \mathbb{Q}[\sqrt[2]{D}]$, D is the discriminant of the equation) (2) In the case $n = 3$, we can find solutions in the successive extensions of binary extension $\mathbb{Q}[\sqrt[2]{D}]$ and the third extension: $\sqrt[2]{\ } \to \sqrt[3]{\ }$ (Cardano method). (3) In the case $n = 4$, we can find solutions in the successive extensions of binary extension $\sqrt[2]{\ }$ and the third extension and furthermore, two binary extensions (Ferrari method):

$$\sqrt[2]{\ } \to \sqrt[3]{\ } \to \sqrt[2]{\ } \to \sqrt[2]{\ } \to \quad (\textbf{BTBB} - \textbf{structure of the solutions}).$$

(Part 2) We introduce binary and ternary extensions for generation of general objects:

$$\begin{array}{ccc} & A & \\ \swarrow & & \searrow \\ Q & & Q^* \end{array} \qquad\qquad \begin{array}{ccc} & A & \\ \swarrow & \downarrow & \searrow \\ Q & Q^* & Q^{**} \end{array}$$

Then we can find the BTBB-structures in evolutions in the universe systematically.

(1) (Physics) photons $\to$ quarks and anti-quarks $\to$ colors of quarks $\to$ quark family
(2) (Biology) RNA $\to$ DNA $\to$ protein $\to$ duplications $\to$ mutations
(3) (Linguistics) automaton sentences$\to$ context free sentences $\to$ context sensitive sentences $\to$ general sentences.

(Part 3) Analogously to quantum field theory, we can realize evolutions in the following manner:

Quantum field theory		Evolution theory
The vacuum $\|0>$	$\to$	The origin $\|0>$
creation operator/annihilation operator		creation operator/conjugate operator

By this scheme we can introduce the concept of non-commutative algebra.

(Part 4) Finally, we make an application to hypercomplex analysis. We can treat Clifford algebra in terms of non-commutative binary extension and the Sylvester algebra in terms of ternary extensions. We put $\mathbf{j}^3 = \mathbf{1}$. The algebra which is generated by

$$Q_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 0 & \mathbf{j}^2 & 0 \\ 0 & 0 & \mathbf{j} \\ 1 & 0 & 0 \end{pmatrix}, \quad Q_3 = \begin{pmatrix} 0 & \mathbf{j} & 0 \\ 0 & 0 & \mathbf{j}^2 \\ 1 & 0 & 0 \end{pmatrix}$$

is called ternary Clifford algebra or Sylvester-Kerner algebra. Putting

$$R_1 = \{x_1Q_1 + x_2Q_2 + x_3Q_3 : x_1, x_2, x_3 \in \mathbb{R}\}, \quad R_2 = \{x_1Q_1 + x_2\mathbf{j}^2Q_2 + x_3\mathbf{j}Q_3 : x_1, x_2, x_3 \in \mathbb{R}\},$$
$$R_3 = \{x_1Q_1 + x_2\mathbf{j}Q_2 + x_3\mathbf{j}^2Q_3 : x_1, x_2, x_3 \in \mathbb{R}\},$$

we can obtain an example of ternary non-commutative Galois extension:

$$\begin{array}{ccc} & \mathbb{R}[x_1, x_2, x_3] & \\ / & | & \backslash \\ R_1 & R_2 & R_3 \end{array} \qquad R_1R_2R_3 = x_1^3 + x_2^3 + x_1^3 - 3x_1x_2x_3.$$

Joint work with: Maria Ławrynowicz, Małgorzata Nowak-Kępczyk

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Atomistic spin model simulations of magnetic and magneto-electric properties of ferromagnetic (Ga,Mn)N

Abstract.

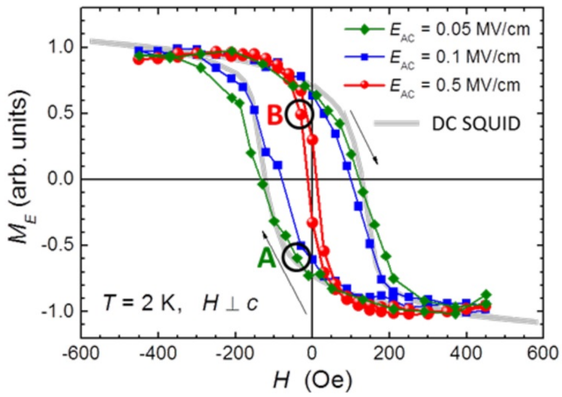

Fig. 1: *Magnetic field H dependence of the normalized amplitude of the sample's magnetic moment induced by an acoustic-frequency sinusoidal electric field E_{AC} at $T = 2$ K measured along the magnetic easy axis.*

An electric field-control of magnetic anisotropy is a promising candidate for building new device functionalities with ultra-low energy consumption. We want to report on this effect in $Ga_{1-x}Mn_xN$-based structures, in which the magnitude and the sign of magnetic anisotropy can be changed by appropriate material engineering and by an electric field. Recently, we have shown that a strong coupling between piezoelectricity and magnetism exists in paramagnetic $Ga_{1-x}Mn_xN$ with $x = 0.025$ [1]. In these piezoelectric systems a voltage V applied across the crystal elongates it and thus deforms the crystal field which surrounds the magnetic ions and so modifies their magnetic anisotropy. Now, by using molecular beam epitaxy grown structures we investigate the magnetoelectric effect in $Ga_{1-x}Mn_xN$ with $x = 0.06$ at $T = 2K$ in the ferromagnetic state. We observe an electric field-induced decrease of coercivity, as shown in the figure, and a non-reversible magnetization switching for magnetic fields close to the coercive field. The data is precisely modeled using atomistic spin approach applied to an ensemble of about 10000 Mn spins within the frame of Landau-Lifshitz-Gilbert description of the precessional motion of magnetization. In numerical simulations we allow for the electric field control of the magnitude of uniaxial (along the wurtzite c axis) anisotropy parameter KU, that in turn affects the effective magnetic field Heff acting on Mn ions. Our results open the route for the repetitive magnetization switching in $Ga_{1-x}Mn_xN$ thin films using sub-nanosecond voltage pulses.

Keywords and phrases: Accelerator, data augmentation, edge computing, FMCW, HCI, gesture recognition, neural networks, DNNs, optimization, radar, intel NCS2

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Depth control for biomimetic and hybrid unmanned underwater vehicles

Abstract. Unmanned underwater vehicles which use biomimetic mechanisms are becoming increasingly useful in the realisation of tasks requiring silent and efficient propulsion. Complex fish kinematics are simplified to some extent and implemented in such vehicles. One of the essential fish behaviours is their ability to adjust their buoyancy using a swim bladder. This paper covers the issues concerning the implementation of artificial swim bladders as well as depth regulators in two underwater vehicles: biomimetic and hybrid. The control of vehicle depth through buoyancy change was examined in the computer simulation and in the experiment. Two types of artificial swim bladder were tested – a rigid cylinder with a piston and an elastic container with a water pump.

Keywords and phrases: BUUV, HUUV, depth control, artificial swim bladder

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$Sp(n)$-orbits of isoclinic subspaces in the real Grassmannians.

Abstract. In the framework of the study of the $Sp(n)$-orbits in the real Grassmannian $G^{\mathbb{R}}(k,4n)$ of k-dimensional non oriented subspaces of a real $4n$-dimensional vector space V, here we consider the case of the isoclinic subspaces whose set we indicate with $\mathcal{IC}$. Endowed V with an Hermitian quaternionic structure $(\mathcal{Q},<,>)$, a subspace U is isoclinic if for any compatible complex structure $A\in\mathcal{Q}$ the principal angles of the pair (U,AU) are all the same, say θ^A. We will show that, fixed an admissible hypercomplex basis (I,J,K), to any such subspace U we can associate two set of invariants, namely a triple (ξ,χ,η) and a pair (Γ,Δ) where Γ itself is a function of (ξ,χ,η). We prove that the angles of isoclinicity $(\theta^I,\theta^J,\theta^K)$ together with (ξ,χ,η,Δ) determine its $Sp(n)$-orbit. In particular if $\dim U=8k+2$ or $\dim U=8k+6$ with $k\geq 0$ the last set reduce to the pair $(\xi=\pm 1,\chi=\pm 1)$.

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Statistics of topological defects in nanostructures based on the Kibble-Zurek Mechanism

Abstract. In this work, the effect that size has over the appearance of topological defects in nanostructures. Based on the Kibble-Zurek mechanism, we will calculate the defect density that arises for a complex system of atoms (a chain and then a strip) by modeling their free energy as a system of Langevin equations and solving the equations of motion, calculation the final state a chain has after a phase transformation of the second order. The system has the form:

$$X_n''[t] + hX_n'[t] + cX_n[t] + V(X_n[t]) + z[t] = 0,$$

where the potential is:

$$V(r) = (-l_c((t/t_q) - l_c))r + 2r^3.$$

For a number of sizes and for different environmental parameters we calculated the average number of topological defects as well as each distribution. The system sizes were that of a nano-structure, 3 to 50 atoms and then up to 500, with a step of 50. We study those results and compare them. We also recreated the simulation for two neighbouring chains of atoms and studied whether or not our previous ndings are also applicable and if a different factor is playing a role for those structures.

Joint work with: Hariton Polatoglou

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Mean periodicity on the Cayley projective plane

Abstract. Let $\mathbb{C}a$ be the algebra of octaves, and let $\mathbb{A}l$ be the collection of all third-order Hermitian octave matrices with the Jordan multiplication

$$A \circ B = \frac{AB + BA}{2}.$$

We denote by $\mathbb{P}^2(\mathbb{C}a)$ the set of all primitive idempotents of the algebra $\mathbb{A}l$, i.e.,

$$\mathbb{P}^2(\mathbb{C}a) = \{X \in \mathbb{A}l \colon X^2 = X, \ \mathrm{Trace}\, X = 1\}.$$

The set $\mathbb{P}^2(\mathbb{C}a)$ can be equipped with a Riemannian structure g such that the Riemannian space $(\mathbb{P}^2(\mathbb{C}a), g)$ is isometric to the Cayley projective plane $F_4/\mathrm{Spin}\,(9)$ [1, Chap. 2]. We present results related to the theory of mean periodic functions on the space $(\mathbb{P}^2(\mathbb{C}a), g)$. In particular, we discuss transmutation transformations of convolution operators, as well as local problems of spectral analysis and spectral synthesis [1], [2].

Keywords and phrases: octaves, mean periodicity, transmutation operators, spectral synthesis

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Inequalities for the inner radii of non-overlapping symmetric domains

Abstract. Let $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be the Riemann sphere, and $r(B, a)$ be the inner radius of the domain $B \in \overline{\mathbb{C}}$ with respect to the point $a \in B$. The inner radius of the domain B is associated with the generalized Green function $g_B(z, a)$ of the domain B by the relations

$$g_B(z, a) = -\ln|z - a| + \ln r(B, a) + o(1), \quad z \to a,$$

$$g_B(z, \infty) = \ln|z| + \ln r(B, \infty) + o(1), \quad z \to \infty.$$

The system of non-overlapping domains is called a finite set of an arbitrary domains $\{B_k\}_{k=0}^{n}$, $n \in \mathbb{N}$, $n \geq 2$ such that $B_k \subset \overline{\mathbb{C}}$, $B_k \cap B_m = \emptyset$, $k \neq m$, $k, m = \overline{0, n}$. Consider the following system of points $A_n := \{a_k \in \mathbb{C}, k = \overline{1, n}\}$, $n \in \mathbb{N}$, $n \geq 2$, satisfying the conditions $|a_k| \in \mathbb{R}^+$, $k = \overline{1, n}$ and $0 = \arg a_1 < \arg a_2 < \cdots < \arg a_n < 2\pi$. Denote by $P_k = P_k(A_n) := \{w : \arg a_k < \arg w < \arg a_{k+1}\}$, $a_{n+1} := a_1$, $\alpha_k := \dfrac{1}{\pi} \arg \dfrac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1, n}$, $\sum\limits_{k=1}^{n} \alpha_k = 2$.

Consider the following problem.

Problem. Let $\gamma \in (0, n]$, $n \in \mathbb{N}$, $n \geqslant 2$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$ $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0, n}$, where $B_0, \ldots, B_n$ are pairwise non-overlapping domains and $B_1, \ldots, B_n$ are symmetric domains with respect to the unit circle. Find the exact upper bound of the product

$$I_n(\gamma) = r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k).$$

This problem has a solution only if $\gamma \leqslant n$ as soon as $\gamma = n + \epsilon, \epsilon > 0$, the problem has no solution. Currently it still unsolved in general, only partial results are known.

Theorem 1. *Let $n = 2$, $\gamma \in (1; 1, 1]$. Then for any different points of a unit circle a_1, a_2 and $a_0 = 0$, and for any different system of non-overlapping domains B_0, B_1, B_2, such that $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, $a_1 \in B_1 \subset \mathbb{C}$, $a_2 \in B_2 \subset \mathbb{C}$, where the domains B_1, B_2, have symmetry with respect to the unit circle, the following inequality holds*

$$r^{\gamma}\left(B_0, 0\right) r\left(B_1, a_1\right) r\left(B_2, a_2\right) \leqslant 4 \frac{\left(\frac{\gamma}{2}\right)^{\frac{\gamma}{2}}}{\left(1-\frac{\gamma}{2}\right)^{1+\frac{\gamma}{2}}} \left(\frac{1-\frac{\sqrt{2\gamma}}{2}}{1+\frac{\sqrt{2\gamma}}{2}}\right)^{\sqrt{2\gamma}}.$$

Equality in this inequality is achieved when a_0, a_1, a_2, and B_0, B_1, B_2, are, respectively, poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{\gamma w^4 + 2(4-\gamma)w^2 + \gamma}{w^2(w^2-1)^2}\, dw^2.$$

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Mathematical approaches and structures in photonic systems

Abstract. First, we will provide a brief review of powerful mathematical tools widely used in photonics. Then, we will discover three impactful results in different areas of photonics. Importantly, these results have been achieved with relatively simple starting mathematical insights.

1) Snell's and Fresnel's laws rigorously describe the reflection and refraction of plane electromagnetic waves at planar interfaces. However, in real world we deal with the beams, which could be represented as a set of plane waves with different wavevectors. The optical beams experience the longitudinal (in-plane) and transverse (out-of-plane) shifts after interaction with planar interfaces. These phenomena are well-known for decades as Goos-Hänchen and Imbert-Fedorov shifts and studied by thousands of works. However, several experiments have recently shown the anomalous enhancement of the photonic spin Hall effect (partial case of Imbert-Fedorov shift) for hyperbolic metamaterials at near-normal angles of incidence ($\approx 0.1°$). Surprisingly, the standard theory of beam shifts fails to explain this phenomenon. In our work [1], we extend the standard theory of optical beam shifts by considering the special case of near-normal beam incidence and show several outstanding features not discussed before to the best of our knowledge.

2) Remote collection and analysis of light is highly important for a plethora of applications including spectroscopy, endoscopy, biosensing, quantum communications, etc. Commercial optical fibers are the best platform for this purpose due to their ability to operate in strongly limited and closed spaces (particularly, in-vivo) in a wide range of environments and external conditions. The payback of this advantage is the low coupling of the incident light into fiber under oblique incidence. Practically, the efficient light coupling into fiber modes is possible only for the incident angles less than 10 degrees. We propose to overcome this fundamental limitation by upgrading the fiber tips with axial-symmetric nanostructure. Namely, we demonstrate the enhancement of the light in-coupling efficiency by several orders of magnitude for the fibers empowered with coaxial silicon nitride and polymer nanorings [2].

3) Babinet's principle is a classical phenomenon in diffraction optics relating the field patterns from scattered on the mutually inverted metallic sheets. We bring this principle to the metasurfaces which are two-dimensional periodic array of subwavelength scatterers (antennas), and engineer so-called a self-complementary metasurface. This type of structure exhibits a constant frequency-independent determinant of the effective surface impedance tensor leading to a set of near-field intriguing properties. Namely, we investigate both theoretically and experimentally the localized energy transfer along an ultranarrow line, and frequency-independent propagation of surface waves with hyperbolic dispersion and constant polarization [3].

Keywords and phrases: photonics, optics, metamaterials, optical beams, fibers, plasmonics, diffraction.

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Estimates of products of some powers of inner radii for multiconnected domains

Abstract. Let $\mathbb{N}$ and $\mathbb{R}$ be the sets of natural and real numbers, respectively, $\mathbb{C}$ be the complex plane, and let $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$. Let $r(B,a)$ be the inner radius of the domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$.

Problem 1. *Let $n \in \mathbb{N}$, $n \geq 2$, a_k, $k = \overline{1,n}$ - some fixed points of the complex plane. Find the maximum of expression $\prod\limits_{k=1}^{n} r(B_k, a_k)$, where B_k, $k = \overline{1,n}$ are pairwise non-overlapping domains in $\overline{\mathbb{C}}$, and $a_k \in B_k$, $k = \overline{1,n}$.*

For $n = 2$ this problem was solved by M.A. Lavrent'ev in 1934, for $n = 3$ — by G.M. Goluzin in 1951, for $n = 4$ — by G.V. Kuz'mina in 1980.

Theorem.[1] *Let $n \in \mathbb{N}$, $n \geq 2$, a_k, $k = \overline{1,n}$ - some fixed points of the complex plane. Then for any pairwise non-overlapping domains B_k, where $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1,n}$ the inequality holds*

$$\prod_{k=1}^{n} r(B_k, a_k) \leq (n-1)^{-\frac{n}{4}} \left(\prod_{1 \leq p < k \leq n} |a_p - a_k| \right)^{\frac{2}{n-1}}.$$

Problem 2. *Let $n \in \mathbb{N}$, $n \geq 2$, a_k, $k = \overline{1,n}$ - some fixed points of the complex plane, γ_k, $k = \overline{1,n}$ are some positive real numbers. Find the maximum of expression $\prod\limits_{k=1}^{n} r^{\gamma_k}(B_k, a_k)$, where B_k, $k = \overline{1,n}$ are pairwise non-overlapping domains in $\overline{\mathbb{C}}$, and $a_k \in B_k$, $k = \overline{1,n}$.*

This problem was formulated by N.A. Lebedev in 1975.

Theorem.[2] *Let $n \in \mathbb{N}$, $n \geq 2$, a_k, $k = \overline{1,n}$ - some fixed points of the complex plane, γ_k, $k = \overline{1,n}$ are some positive real numbers and $\lambda := \sum\limits_{k=1}^{n} \gamma_k$. Then for any pairwise non-overlapping domains B_k, where $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1,n}$ the inequality holds*

$$\prod_{k=1}^{n} (r(B_k, a_k))^{\gamma_k} \leq (n-1)^{-\frac{\lambda}{2}\frac{n-1}{n-2}} \frac{\prod\limits_{i,j=1, i<j}^{n} |a_j - a_i|^{\frac{2(\gamma_i+\gamma_j)}{n-2}}}{\left(\prod\limits_{k=1}^{n} r(B_k, a_k)\right)^{\frac{\lambda}{n-2}}}.$$

Keywords and phrases: inner radius of domain, non-overlapping domains, Green's function.

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Exact solution of ferromagnetic 3D Ising model and computational complexity of spin-glass 3D Ising system

Abstract. In this talk, I would like to introduce the advances in the 3D Ising models. Two conjectures are introduced for solving explicitly the ferromagnetic 3D Ising model [1]. An additional rotation in the fourth dimension and the weight factors on the eigenvectors, are proposed to deal with the topologic problem of the ferromagnetic 3D Ising model. With the Clifford algebra representation, four theorems (Trace Invariance Theorem, Linearization Theorem, Local Transformation Theorem, Commutation Theorem) are proved to give a positive answer to the two conjectures [2], so that the exact solution of the ferromagnetic 3D Ising model derived in [1] is correct. Furthermore, a method of Riemann-Hilbert problem is employed for Zhang's conjectures in a ferromagnetic 3D Ising model, regarding to trivialization of topological structure [3] and topological phases [4]. The exact solution of the 2D Ising model with a transverse field is derived by a mapping between the two models [5]. The computational complexity of the spin-glass 3D Ising model is determined to be subexponential and superpolynomail [6].

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Harmonic measure for the fractional Laplacian and its applications

Abstract. The talk deals with the theory of potentials with respect to the α-Riesz kernel $|x-y|^{\alpha-n}$ on $\mathbb{R}^n$, $n \geqslant 2$, where $0 < \alpha \leqslant 2$ and $\alpha < n$, associated with the fractional Laplacian $\Delta^{\alpha/2}$. The results discussed have been established in [10].

We first focus on the inner α-harmonic measure ε_y^A for arbitrary $A \subset \mathbb{R}^n$, being motivated by the known fact that this concept is the main tool in solving the generalized Dirichlet problem for α-harmonic functions (see e.g. [1, 8]). Here ε_y denotes the unit Dirac measure at $y \in \mathbb{R}^n$, and μ^A the inner α-Riesz balayage of a Radon measure μ to $A \subset \mathbb{R}^n$ (see [9], cf. also [5] where $\alpha = 2$).

We describe in particular the support of the inner α-harmonic measure ε_y^A, provide a formula for evaluation of its total mass $\varepsilon_y^A(\mathbb{R}^n)$, prove the vague continuity of the map $y \mapsto \varepsilon_y^A$ outside the inner α-irregular points for A, and obtain necessary and sufficient conditions for ε_y^A to be of finite energy (more generally, for ε_y^A to be absolutely continuous with respect to inner capacity) as well as for the identity $\varepsilon_y^A(\mathbb{R}^n) \equiv 1$ to hold. Those criteria are given in terms of the newly defined concepts of inner α-thinness and inner α-ultrathinness of A at infinity (see [10]), which in the case where $n \geqslant 3$, $\alpha = 2$, and A is Borel coincide with the well known concepts of outer 2-thinness at infinity introduced by J.L. Doob [6] and M. Brelot [2, 3], respectively.

Further, we extend some of these results to μ^A general by means of establishing the integral representation formula for inner balayage:

$$\mu^A = \int \varepsilon_y^A \, d\mu(y),$$

and we also show that the inner balayage to A arbitrary can always be reduced to balayage to Borel sets. In more details, for every $A \subset \mathbb{R}^n$, there exists $A_0 \subset A$ represented as a countable union of compact sets and such that

$$\mu^A = \mu^{A_0} \quad \text{for all } \mu.$$

As an application of the latter theorem, we prove the vague and strong continuity of the inner swept, resp. inner equilibrium, measure under an approximation of A arbitrary, thereby strengthening B. Fuglede's result on the continuity of the inner equilibrium measure [7, Theorem 4.2], given for A Borel.

Being mainly new even for $\alpha = 2$, the results obtained also present a further development of the theory of inner Newtonian capacities and of inner Newtonian balayage, originated by H. Cartan [4, 5].

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